Analysis of modelling water quality in lakes and reservoirs

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Abstract

Water quality in lakes is analyzed in this study. It deals with modeling of long residence times in water bodies, natural or man-made. The key driving force of Lake Dynamics is usually temperature. Its vertical distribution defines whether a lake is stratified or not. The result shows that runoff process is involved the biochemical oxygen demand concentration of the lake water is reflected mainly by the subsurface runoff component, suspended solid (ss) concentration is affected mainly by the surface runoff component.

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Introduction

(Cho and Seo, 2007a) assumed that the runoff pollution loads, at the time of raining from the adjacent resort and farmland located at the watershed, affect the lake water quality significantly. Recently, a number of studies were conducted with respect to the characteristics of such-like pollution loads by rainfall runoff (Bertrand-Krajewski et al., 1998; Bedient et al., 1978). Since the first flushing by the initial rainfall is considerable and takes a large portion in the total runoff of pollution loads, especially in urban areas, research regarding this has been actively conducted (Lee and Bang, 2000; Lee et al., 2003; Novotny, 2003). Based on the storm water runoff data from the Yeongrang Lake watershed, Cho and Seo (2007b) prepared dimensionless M(V) curves indicating the distribution of pollutant mass vs. volume in storm water discharges and carried out the regression analysis on the dimensionless M(V) curves with the power function and second and third degree polynomial equation curves, in order to develop a tool that allows quantitative evaluation of first flushing. They have also deducted rainfall-runoff curves and rainfall-runoff pollutant load curves by investigating the rainfall runoff from Jangcheon, which provides most of the runoff input, as well as the storm sewers that have a large amount of rainfall runoff, and by estimating the runoff and the runoff pollution loads of each rainfall. The WASP model (Ambrose, 1987; James et al., 1995; Kim et al., 2004; Lung et al., 1993; Lung and Larson, 1995; Tufford and McKellar, 1999; US EPA, 2006; Wool et al., 2001), developed by the U.S. EPA, is widely used domestically and internationally as the model to calculate the water quality and phytoplankton in a lake. Recently, Zhang et al. (2008) have developed the LM2-Toxic model, based on the WASP model, to calculate PCB concentration, and they have applied it to Lake Michigan. Canu et al. (2004) have combined EUTRO, the ecological module of WASP, with a two-dimensional finite element hydrodynamic model, and then applied it to a lagoon in Venice. In this study, we have applied the WASP model to the water quality control of Yeongrang Lake, which is now deteriorated. By using the rainfall-runoff curves, as well as the rainfall-runoff pollutant load curves.

Methods and material

The structure and characteristics of WASP7 model

The WASP model is widely used in interpreting and predicting the water quality response to the pollution of water resources by natural phenomena or human activities. The WASP is a dynamic model for a water body including sediment, allowing one-, two-, and three-dimensional calculations, and composed of an eutrophication module that enable calculation of the 13 variables related to water quality, a toxicant module, a mercury module, and a heat module, as shown. The model can be used in connection with the pollution load data calculated by external models, such as SWMM, HSPF, and NPSM, using the preprocessor, or it can organize the input data in connection with hydrodynamic models, such as EFDC and DYNHYD. In addition, input data, such as various constants, parameters, segments, loads, and boundary conditions, can be easily organized. It also deals with the change of advection, dispersion, point source, and diffuses pollution loads depending on time. In the postprocessor, the calculated water quality data can be displayed with the measured data, so that they can be easily calibrated.

The relationships between the runoff components and water quality changes are finally concluded. The study shows the Biochemical Oxygen Demand (COD) concentration of the lake water is affected mainly by the sub-surface runoff component, and on the contrary, Suspended Solid (SS) concentration is affected mainly by surface runoff component.
Runoff simulation
The Xinanjiang runoff model is used in this study to simulate the runoff process. This model has been successfully and widely applied in humid and semi-humid areas in China since its development in the 1970s (Zhao et al., 1992). It is a semi-distributed runoff model for use in humid and semi-humid regions. The whole catchment is divided into several sub-catchments according Thiessen method. The runoff generation in each sub-catchment is simulated, and routed to the catchment outlet by using of the Muskingum method.

Runoff generation model
The main feature of the Xinanjiang model is the concept of runoff generation on repletion of storage, which means that runoff is not generated until the soil moisture content of the aeration zone reaches maximum capacity, and thereafter runoff equals the rainfall excess without further loss. This concept of runoff generation can be expressed in Fig. 2 (the shaded box represents a soil column).

In Fig. 2, P, E and Fc are precipitation (mm/day), evapotranspiration (mm/day) and final infiltration rate (mm/day) in specific time interval, respectively. W is areal mean tension water storage (mm) and WM is the capacity of W. RS and RG is surface runoff and ground water contribution respectively. According the assumption of this concept, by using the water balance calculation, we have:

Before repletion of storage: \( P - E = W_2 - W_1 \)  
(1a)

After repletion of storage: \( P - E - R = W_M - W_1 \)  
(1b)

Where, \( W_1, W_2 \) the areal mean tension water storage at beginning and end of specific time interval. R-the total runoff generation in specific time interval, and \( R = RS + RG \).

Therefore, the total runoff generation can be separated as two parts:

\( RS = P - E - Fc \); \( RG = Fc \)  
(2)

Strictly speaking, equation (1) and (2) are only suitable to a specific point in the catchment. Because the spatial and temporal distribution of tension water storage is not possibly uniform in the whole sub-catchment area. The Xinanjiang model uses a parabolic curve to represent the distribution variation of tension water storage, which showed in equation (3) and Fig. 3:

\( W = WM \)  

Fig. 1. Conceptual model of heat budget of a Lake.

Fig. 2. The concept of runoff genera.

Fig. 3. Distribution of tension water storage capacity.
Where, \( f \) - the partial pervious area of the catchment whose tension water capacity is less than or equal to the value of the ordinate \( W'_{m} \). The tension water capacity at a point \( W'_{m} \) varies from 0 to its maximum \( W'_{mm} \). F is total catchment area and B is exponent of the tension water capacity curve, which represents the non-uniform of tension water storage distribution (Figure 3). Therefore, the areal mean tension water capacity \( WM \) can be expressed by integration:

\[
WM = \int_{0}^{1} W_{m} \cdot \left( \frac{f}{F} \right) = \frac{W'_{mm}}{1 + B} \tag{4}
\]

And the ordinate which corresponding to \( W \) is:

\[
A = W_{mm} \left[ 1 - (1 - (1 - W/WM)^{1/B}) \right] \tag{5}
\]

The runoff generation can be expressed by following equations:

when \( P - E \leq 0 \), \( R=0 \) (no runoff generation);
when \( P - E > 0 \), \( R > 0 \) (runoff generates):

\[
R = P - E - WM + W + WM \left[ 1 - (P - E + A) / W'_{mm} \right]^{1/B} \tag{6a}
\]

when \( P - E + A \geq W'_{mm} \) \( R = P - E - \left( WM - W \right) \), when \( P - E + A < \) \( W'_{mm} \) \( \tag{6b} \)

The result of above generation equation is total runoff generation, which including surface runoff and sub-surface runoff. Because the variation of runoff generation area is considered, the runoff component separation equation (2) must be changed to equation (7):

When \( P - E + Fc \):

\[
RG = Fc \times (f/F) = Fc \times (R/(P-E)) \tag{7a}
\]

\[
RS = (P - E + Fc) \times (f/F) = R - RG \tag{7b}
\]

When \( P - E \leq Fc \):

\[
RS = 0, \quad RG = R \tag{7c}
\]

The equation (6) and (7) are the so-called the Xinanjiang model with two runoff components. Actually, the surface runoff \( RS \) separated from equation (2) and (7) usually includes most of interflow part. Strictly speaking, it should be called as direct runoff and includes two parts: the “real” surface flow and interflow. In the three runoff components’ Xinanjiang model, the total runoff generation is separated into three parts: \( RS \)-surface runoff, \( RI \)-a contribution to interflow and \( RG \)-the contribution to the groundwater by introducing another 4 parameters instead of final infiltration rate \( Fc \). They are: \( S \)-free water storage (mm), \( SM \)-areal mean free water storage capacity (mm), \( KI \)-coefficient related to interflow component and \( KG \)-coefficient related to groundwater component. The water component equation can be expressed as follows:

\[
RG = KG \times S \times FR \tag{8a}
\]

\[
RI = KI \times S \times FR \tag{8b}
\]

\[
RS = 0, \text{ when } P - E + S \leq SM \tag{8c}
\]

\[
RS = (P - E + S - SM) \times FR, \text{ when } P - E + S > SM \tag{8d}
\]

\[
R = RS + RI + RG \tag{8e}
\]

where, \( FR \) represents the area where runoff generates.

**Evapotranspiration model**

In the Xinanjiang model, a three-layer model is used to estimate the evapotranspiration depending on the pan evaporation. The soil column is divided into three layers called upper, lower and deepest layer, which having areal mean tension water storage \( WU \), \( WL \) and \( WD \) respectively \((WU+WL+WD=W)\). The tension water storage capacity \( WUM \), \( WLM \) and \( WDM \) respectively \((WM=WUM+WLM+WDM)\). The evapotranspiration firstly begins from the upper layer, and after the \( WU \) is exhausted, evapotranspiration starts from lower layer. The actual evapotranspiration in the upper and lower layer \( EU \), \( EL \) can be expressed as follows:

\[
EU = K \times EM, \text{ when } WU > EM \tag{9a}
\]

\[
EU = WU, \text{ when } WU \leq EM \tag{9b}
\]

\[
EL = (K \times EM - EU) \times WL/WLM \text{ when } EL \geq C \times EM \tag{9c}
\]

\[
EL = C \times EM, \text{ when } EL < C \times EM \text{ and } WL \geq C \times EM \tag{9d}
\]
Where, K-coefficient to adjust evaporation; EM-pan evaporation; C-minimum evapotranspiration efficiency, which is a constant in a specific basin area.

When the tension water storage in lower layer WL is decreased to a value of C×EM, the model assumes that evapotranspiration in the deepest layer (ED) begins at following rate:

\[ \text{EL} = \text{WL, when } \text{WL} < C \times EM \]  
\[ \text{ED} = C \times (K \times EM - \text{EU}) - \text{EL} \text{ when } \text{WL} < C \times EM \]  

**Estimation of potential evapotranspiration**

The Xinanjiang model includes two inputs: the measured areal mean precipitation and measured pan evaporation on the sub-area. Because there is no pan evaporation data set in Japan, in this study the potential evapotranspiration data is used instead of pan evaporation data. Usually it is difficult to estimate the potential evapotranspiration, because evapotranspiration is dependent upon some parameters, which is difficult to estimate their value. This study uses Hamon equation to estimate potential evapotranspiration:

\[ E_p = 0.14D_0^2 p_t \]  

Where, \( E_p \)-daily average potential evapotranspiration (mm/day); \( D_0 \)-available sunshine duration (12hrs/day); \( p_t \)-saturated absolute humidity (g/m\(^3\)).

Actually the saturated absolute humidity is the density of water vapor (kg/m\(^3\)), i.e., mass of water vapor in unit volume of air under saturation vapor pressure. It can be calculated by using following equation:

\[ p_t = 216.7 \frac{\varepsilon_{\text{sat}}}{T} \]  

Where, \( \varepsilon_{\text{sat}} \)-saturation vapor pressure (hPa); T-air temperature (K); \( p_t \)-in g/m\(^3\).

Then saturation vapor pressure can be approximately calculated by Tetens equation:

\[ \varepsilon_{\text{sat}} = 6.1078 \times 10^{\frac{7.5T}{237.3+T}} \]  

**Optimization of parameters and runoff simulation**

The main parameters of runoff generation model, evapotranspiration model and runoff separation equation can be summarized as following:

Runoff generation: WM, B

Evapotranspiration: K, WUM, WLM, C

Runoff Separation: SM, KG, KI

K is a coefficient related to pan evaporation. Among above parameters, the simulation result is particularly sensitive to this parameter which controls the water balance of whole catchment. This parameter should be determined by field experiment. Due to lack of pan evaporation data, this study firstly tries to find suitable K value for the Kamafusa Lake catchment by optimization. The parameter SM, KG and KI are also more sensitive to output and optimized according to the recommended value for humid area. For other parameters like WUM, WLM, B and C, the values recommended for humid area in literatures are adopted.

The two indexes are used to evaluate the simulation results of each year:

\[ \text{WBD} = \frac{\sum \text{Q}_{\text{obs}} - \sum \text{Q}_{\text{calc}}}{\sum \text{Q}_{\text{obs}}} \times 100\% \]  
\[ \text{NTD} = 1 - \frac{\sum (\text{Q}_{\text{obs}} - \overline{\text{Q}}_{\text{calc}})^2}{\sum (\text{Q}_{\text{obs}} - \text{Q}_{\text{calc}})^2} \]  

Where, WBD-annual water balance difference (%); NTD-Nash index, which means the value is closer to 1, the better result is simulated; \( \text{Q}_{\text{obs}} \)-observed runoff flow rate (m\(^3\)/s); \( \text{Q}_{\text{calc}} \)-calculated runoff flow rate (m\(^3\)/s) and \( \overline{\text{Q}}_{\text{calc}} \)-daily average calculated runoff flow rate (m\(^3\)/s).

The mathematical modeling of lakes is based on the following equations:
Conservation of mass
Conservation of momentum
Transport of contaminants
Chemical and biological process kinetics
Conservation of heat

The first two equations are the foundations of all hydrodynamic problems, while the third is used in the water environment and other environments (air, soil, and biota). The fifth, the conservation of heat, has not been used so far in this text and while it may be applicable to the air environment, it is usually not of relevance to rivers.

Usually lakes require to be modelled when there are problems with water quality and most often occur in both shallow and deep lakes when they are stratified. Stratification is when there is a significant temperature difference between the epilimnion (upper waters) and the hypolimnion (lower waters). The narrow band between both with a steep temperature gradient is the thermocline. If the densiometric Froude number is < 0.01, and then the water body stratified. The physical processes involved in the heat budget of a lake are depicted in Fig. 1.

Model setup
One technique to model stratified lakes is to divide the lake into a number of horizontal layers as shown in Fig. 2. The more simplistic model will have three 'layers': the epilimnion, the thermocline and the hypolimnion. More capable models will have several layers. On each layer, a mass balance is computed, allowing for the transfer of flow and constituents from one layer (vertically) to another.

Model performance
For instance, the mass flow balance of the jth (horizontal) layer is
\[ A V_j = Q_{jin} - (Q_j - 1 + Q_j +) = P_{inflow} \]
\[ A \square Q_j \quad \text{Outflow} \]
\[ \square g \quad \text{Groundwater} \]

Where, \( V_j = \) volume of the jth layer
\( Q_{jin} Q_{jout} = \) horizontal surface inflow and outflow (abstraction) from the ith layers
\( Q_j = \) outflow vertically from the jth layer
\( Q_{j-1}, Q_{j+1} = \) inflow vertically to the jth layer from the j - 1 and j + 1 layers

Model application
Several of the above terms require information on the heat budget of the lake as depicted in Fig. 2. Assuming that P, E and Q are included in the vertical fluxes \( \{Q_{i-1}, i \text{ and } g_{i+1}\} \) then the mass conservation for a typical layer of thickness \( Az \) is
\[ A V_j = Q_{jin} - (Q_j - 1 + Q_j +)z - Q_{jout} \]
where, \( Q_{jin}' Q_{jout}' \) = horizontal flow advection
\( Q_{j-1}, Q_{j+1}' \) = vertical (low into or out of the element)

Results
The conservation of heat equation stored in a horizontal control volume layer is described by Orloh (1981). Solution of the mass balance (2) requires information on the heat energy balance. The end result of modelling of reservoir is the production of vertical profiles of temperature to determine the lake status for different inputs of solar heat and freshwater flows.

Detailed descriptions of the above and solution techniques are to be found in several articles (Orioh 1981; James, 1993; Fischer et al., 1979). A computer program DYRESM detailed by (Fischer et al., 1979) and Imberger (University of West Australia) is commercially available to solve lake hydrodynamic and lake water quality problems.

Conclusion
These model are used predict temperature profile in stratified lakes and are ideal for lake water quality...
management in identifying incipient seasonal eutrophication. Complex models may extend to two and three dimension and/or incorporate reaction chemistry and ecological models. Recent work by lake models, DYRESM.

References


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